Journal of Sound and Vibration (2001) **240**(3), 592–597
doi:10.1006/jsvi.2000.3162, available online at http://www.idealibrary.com on IDEAL®

ON THE ACTIVELY CONTROLLED NOISE BARRIER

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(*Received* 19 *January* 2000, *and in final form* 19 *June* 2000)

1. INTRODUCTION

Recently, the active control technique was used to suppress the sound diffracted by a noise barrier [1]. This method operated by the cancellation of the sound pressure at the diffraction edge of the barrier, which normally behaves like the virtual source of the diffracted field. The performance of such control system was investigated using multichannel adaptive signal processing, and the developed theory was further verified by experiments [2]. On the basis of this noise-reduction strategy, Shao *et al.* [3] made an interesting work and two conclusions were drawn: "(1) the model of minimizing the sum of squared acoustic pressures is more effective than the model of cancelling sound pressure [2], (2) the arc-type arrangement of the secondary sources can apparently improve the effectiveness of active control, especially for the model of minimizing the sum of squared acoustic pressures and relatively more secondary sources." We noted that the first conclusion is not truly accurate since the two models come from the same principle of cancelling sound pressure on the edge. On the other hand, more studies need to be carried out to confirm that arc-type arrangement of secondary sources is a better choice compared to the line-type arrangement of secondary sources. It is the purpose of this paper to address the above issues and numerical examples will be provided to show that greater attenuation derived from the arc-type arrangement over the line-type arrangement is the result of relative nearer distance between the secondary sources and the primary source.

2. THEORETICAL MODEL

Physically, the sound pressure at the vicinity of the edge has a dominant effect on the diffracted field, so the practical control strategy is focused on cancellation at multiple points along the diffraction edge. The proposed analytical models in references [1–3] are shown in Figure 1. Using the principle of superposition to write the total vector of sound pressures produced at the n locations as:

$$\mathbf{p} = \mathbf{p}_p + \mathbf{p}_s \quad \text{or} \quad \mathbf{p} = \mathbf{p}_p + \mathbf{Z} \cdot \mathbf{q}_s, \tag{1}$$

where the sound pressure $\mathbf{p}_p$ produced by primary source is an $n \times 1$ complex-valued vector, the secondary strength $\mathbf{q}_s$ is an $m \times 1$ complex-valued vector, and the matrix $\mathbf{Z}$ of complex

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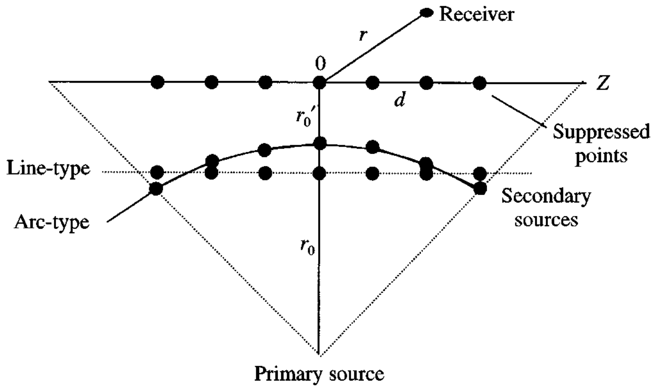

Figure 1. The arrangement of secondary sources.

acoustic transfer impedances is specified by

$$\mathbf{Z} = \frac{j\omega\rho_0}{4\pi}\begin{bmatrix} \frac{e^{-jkr_{11}}}{r_{11}} & \frac{e^{-jkr_{12}}}{r_{12}} & \cdots & \frac{e^{-jkr_{1m}}}{r_{1m}} \\ \frac{e^{-jkr_{21}}}{r_{21}} & \frac{e^{-jkr_{22}}}{r_{22}} & \cdots & \frac{e^{-jkr_{2m}}}{r_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{e^{-jkr_{n1}}}{r_{n1}} & \frac{e^{-jkr_{n2}}}{r_{n2}} & \cdots & \frac{e^{-jkr_{nm}}}{r_{nm}} \end{bmatrix}, \tag{2}$$

where r_{nm} is the distance from the mth secondary source to the nth cancelling point, and $\mathbf{Z}$ is an $n \times m$ matrix with complex entries. Using the vector space concept [4], the sound field to be controlled, $\mathbf{p}$ can be regarded as the residual vector. If we choose the number of secondary sources m to be equal to the number of suppressed locations n, then the matrix $\mathbf{Z}$ is square, and we can get the solution $\mathbf{q}_s = -\mathbf{Z}^{-1}\mathbf{p}_p$ to ensure the vector $\mathbf{p} = 0$, provided that $\mathbf{Z}$ is non-singular. More generally, if such a solution does not exist, we can attempt to find a vector $\mathbf{q}_s$ which minimizes $||\mathbf{p}||$. This is referred to as the least-squares solution. However, there might be many vectors that result in the same minimum value of $||\mathbf{p}||$, be it zero or otherwise. In those cases we again seek the unique x, which is of minimum norm, that is, we also minimize $||\mathbf{q}_s||$. The $\mathbf{q}_s$ that minimizes both of the norms is called the minimum-norm, least-squares solution, or sometimes the minimum least-squares solution.

All of these contingencies are accommodated by the pseudoinverse, or Moore–Penrose inverse, denoted by $\mathbf{Z}^+$, with this, the minimum-norm, least-squares solution is written simply as

$$\mathbf{q}_s = -\mathbf{Z}^+\mathbf{p}. \tag{3}$$

When a unique exact solution is available, the pseudoinverse is the same as the usual inverse, e.g., in the case of $m = n$. Whereas, for the overdetermined system (i.e., there are more cancelling points, n than the number of secondary sources, m) the pseudoinverse is given by

$$\mathbf{Z}^+ = (\mathbf{Z}^H\mathbf{Z})^{-1}\mathbf{Z}^H \tag{4}$$

where the superscript **H** is the Hermitian transpose of a vector. This is very often the case with active control, and the expression is the same as the optimal secondary source strengths in reference [3]. It shows that, the two models in references [1, 3] are in essence not different, but corresponding to the relationship of the number of secondary sources and cancelling points. So the Moore–Penrose inverse equation (4) will be uniformly used in the following numerical computations, regardless of $m = n$ or $m < n$.

In the actively controlled diffracted sound field, the velocity potential at receiver (r, θ) is given as [2, 5]

$$\phi(r, \theta) = \phi_0(r, \theta) + \sum_{i=1}^{m} \mathbf{q}_i \phi_i(r, \theta) \tag{5}$$

where $\mathbf{q}_i$ is the ith entry of column vector $\mathbf{q}_s$, and ϕ_i can be calculated by equation (2) of reference [1]. Then the effectiveness of the active control is defined as

$$\Delta L = 20 \log_{10}(\phi_{off}/\phi_{on}) \tag{6}$$

where ϕ_{off} and ϕ_{on} are, respectively, the value of equation (5) without and with secondary sources.

3. NUMERICAL SIMULATION

To investigate the effectiveness of active control, the same conditions as referenced in references [1, 3] are adopted. By defining r_0 and r'_0 as the distance from the primary source and secondary sources to the barrier, respectively, θ_0 and θ'_0 as the angle between the primary/secondary sources with the barrier and the z is the co-ordinate where cancellation take place. Assume that the primary source is at the point $(r_0, \theta_0, z_0) = (0{\cdot}5\text{ m}, 60^\circ, 0)$ and 5 kHz pure tone is used. For the line-type secondary sources, $r'_0 = 0{\cdot}2$ m and $\theta'_0 = \theta_0$. The receiver at $(r, \theta, z) = (1{\cdot}0\text{ m}, 300^\circ, 0\text{–}2{\cdot}0\text{ m})$, and the intervals $d = 0{\cdot}03$ m (for less than half of the wavelength).

At first, the numbers of secondary sources and suppressed points are assumed to be variable in the case of the arrangements of line- and arc-type secondary sources, and the numerical results are shown in Figure 2. It can be seen that, whether the secondary sources are arranged on the straight line or an arc, an effective and wider sound-reduction range can be achieved by using greater number of suppressed locations than secondary sources. Moreover, for fixed number of secondary sources, with more suppressed points, the cancellation is decreased. It indicates that, for a practical noise barrier, it is possible to seek optimal conditions for active control. First, an equal number of secondary sources and cancelled points are employed, and the attenuations of sound pressure over required location with all the different arrangements of secondary sources are simulated, the optimal number of secondary sources corresponds to the greatest reduction. Second, the number of suppressed points is increased to obtain the optimal number of suppressed points; it corresponds to the widest required control region. In the practical application, the optimal control system is often balanced by minimum expenditure.

For the model adopted in this note, the relationship of sound attenuation with the number of secondary sources is shown in Figure 3a. Due to the secondary sources located symmetrically, the receiver at $(r, \theta, z) = (1{\cdot}0\text{ m}, 300^\circ, 0)$ is considered. Obviously, a unique exact solution to equation (1) is available due to $m = n$, and the performances of the two models are similar and reach their optimum effect when the number of the secondary source is 15 or more. In Figure 3b the simulation results with optimal number $m = 17$ is given. It

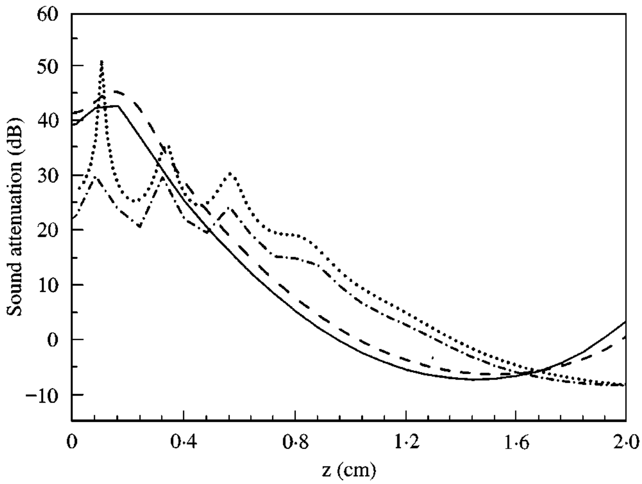

Figure 2. Sound attenuation by active control as a function of z position of receiver in the diffracted sound field. ——, $m = 11, n = 11$, line-type; – – –, $m = 11, n = 11$, arc-type; -·-·-·-·-, $m = 11, n = 23$, line-type; ·········, $m = 11$, $n = 23$, arc-type.

demonstrates that the sound attenuation does not increase with the number of the suppressed points, so we can use a relatively smaller number of secondary sources and measurement positions to achieve noise reduction close to the optimal result. It should be pointed out that, according to the simulations and experiments made by Omoto and Fujiwara [1], active control can be effective when the intervals of the secondary sources (suppressed points) are less than half of the wavelength. In this note, due to the geometrical limit, for the arrangement of arc-type secondary sources, it has the maximum $m = 42$, which is ignored in reference [3]. For low frequencies, the intervals of secondary sources (and suppressed points) will be increased, so the number of arc-type secondary sources will be reduced greatly, this should be noted in the practical applications.

Although the arc-type arrangement seems to be more effective than that of line-type, when the number of secondary sources and suppressed points are chosen properly, the main mechanism is still related to the destructive interference near the diffraction edge caused by wavefront matching [1], because the secondary sources on the arc are relatively nearer to the primary source than those on the straight line. As an example, for the case of $m = 17$, $n = 33$, if the line-type secondary sources are located at the average distance (i.e., the distance of primary to the secondary sources line) of 0·24 m (see Figure 1), or $r'_0 = 0{\cdot}26$ m, the active control is equally effective compared with that of arc-type arrangement as shown in Figure 4. This observation point to the fact that , we can use different arrangements of the secondary sources to produce equally effective cancellation of sound pressure. The only crucial factor that results in greater cancellation of sound pressure is the distance between the secondary and the primary sources.

4. CONCLUSION

The active control of the sound diffracted by a semi-infinite barrier is addressed in this note, and further numerical simulation suggests that, based on the principle of cancelling sound pressure over the noise barrier, using a greater number of suppressed points than the secondary sources can produce smaller but more widespread reductions, and the

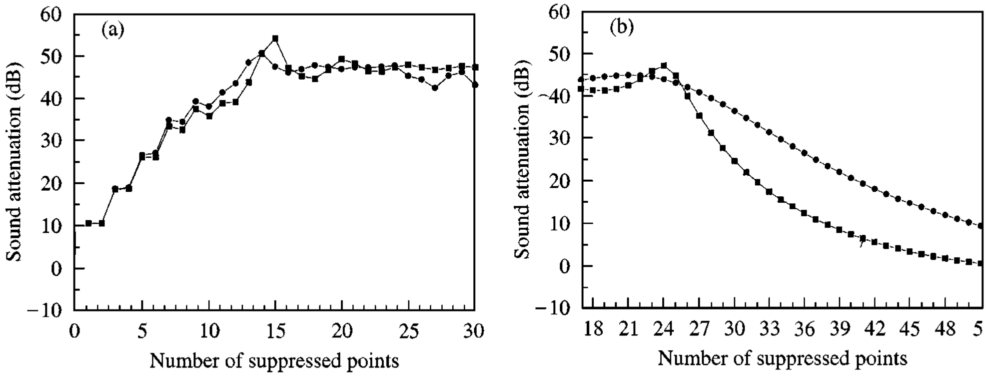

Figure 3. Sound attenuation by active control as a function of number of suppressed points for the receiver at (1·0 m, 300′, 0), with the (a) equal number of secondary sources, $m = n$; (b) fixed number of secondary sources, $m = 17$ and $m < n$. —■—, line-type; —●—, arc-type.

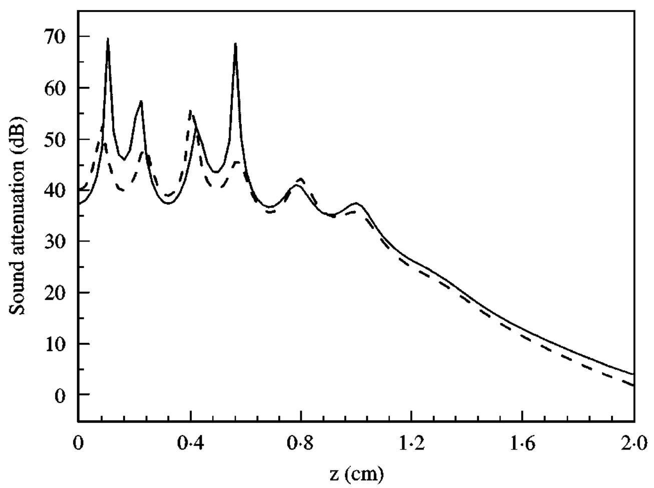

Figure 4. Sound attenuation by active control as a function of z position of reciever in the diffracted sound field. $m = 17$, $n = 33$ and the secondary sources on the line $r'_0 = 0{\cdot}26$ m. – – –, line-type; ———, arc-type.

arrangement of the secondary sources obeys the rule that secondary sources must be located near the primary source to obtain more effective control of diffracted sound field. Moreover, it is possible to determine the optimal number of secondary sources and error sensors (suppressed points) by the numerical simulation. In this note, the optimal reduction can be approximated using a finite number of secondary sources and suppressed points. This is helpful in minimizing the cost of implementation and future practical application of this control strategy.

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